

# Compressibility and collisional effects on thermal instability of a partially ionized plasma in porous medium

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**Abstract** : The compressibility and collisional effects on the thermal instability of a partially ionized plasma in porous medium are considered. The medium permeability and the magnetic field are found to have destabilizing and stabilizing effects respectively on the thermal instability for both stationary convection and overstable cases. A sufficient condition for the non-existence of overstability has been obtained. The magnetic field introduces oscillatory modes in the system which were non-existent in the absence of magnetic field. The effect of compressibility is found to postpone the onset of thermal instability.

**Keywords** : Compressibility, collisions, thermal instability, partially ionized plasma, porous medium

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## 1. Introduction

Chandrasekhar [1] has given a detailed account of thermal instability of fluids under varying assumptions of hydromagnetics. The Boussinesq approximation has been used which is well justified in the case of incompressible fluids.

The equations governing the system become quite complicated, when the fluids are compressible. To simplify the set of equations governing the flow of compressible fluids, Spiegel and Veronis [2] have made the following assumptions :

- (i) The fluctuations in density, pressure and temperature, introduced by the motion, do not exceed their total static variations.
- (ii) The depth of the fluid layer is much smaller than the scale height as defined by Spiegel and Veronis [2].

Under the above approximations, Spiegel and Veronis [2] have found that the flow equations are the same as those for incompressible fluids except that the static temperature gradient is replaced by its excess over the adiabatic one and  $C_v$  is replaced by  $C_p$ . The thermal instability of compressible fluids, under the above approximations, in the presence of magnetic field and rotation has been studied by Sharma [3]. Sharma [4] has also considered the thermal instability of a compressible Hall plasma.

More often than not, the plasma is not fully ionized but may instead be permeated with neutral particles. There are several situations where the interaction between the ionized and neutral gas components becomes important in cosmic physics. It has been reported by Strömberg [5] that ionized hydrogen is limited to certain rather sharply bounded regions in space surrounding, for example, O-type stars and clusters of such stars and that the gas outside these regions is essentially non-ionized. Other examples of the existence of such situations are given by Alfvén's [6] theory of the origin of the planetary system, in which a high ionization rate is suggested to appear from collisions between a plasma and a neutral gas cloud and by the absorption of plasma waves due to ion-neutral collisions such as in the solar photosphere and chromosphere and in cool interstellar clouds [7,8]. The collisional effects on various instability problems have been considered by Hans [9], Rao and Kalra [10] and Sharma and Misra [11]. The medium has been considered to be non-porous in all the above studies.

The physical properties of meteorites, comets and interplanetary dust strongly suggest the importance of porosity in astrophysical context [12]. Sharma and Bhardwaj [13] have considered thermosolutal instability of a partially ionized plasma in porous medium. Sharma and Sunil [14] and Sharma and Neela Rani [15] have studied the thermosolutal instability of a partially ionized Hall plasma in porous medium in presence of a uniform vertical and a uniform horizontal magnetic field respectively. In another study, Sharma and Sunil [16] have considered the Rayleigh-Taylor instability of a partially ionized plasma in porous medium in presence of a variable magnetic field. The compressibility, collisions between ionized and neutral particles and medium porosity effects are likely to be important in astrophysical situations like stellar interiors and atmospheres and in geophysical situations like Earth's molten core. The thermal instability of a partially ionized plasma in porous medium including the effects of collisions and compressibility is, therefore, considered in the present paper.

## **2. The physical problem, formulation and dispersion relation**

Here we consider an infinite, horizontal, compressible and composite plasma layer, consisting of finitely (electrical) conducting ionized component of density  $\rho$  and neutral component of density  $\rho_d$  of thickness  $d$  in porous medium of porosity  $\epsilon$ , and acted on by a uniform vertical magnetic field  $H(0,0,H)$  and gravity field  $g(0,0,-g)$ . This layer is heated from below such that a steady adverse temperature gradient  $\beta(= |dT/dz|)$  is maintained. We assume that both the ionized gas and the neutral gas behave like continuum fluids and that the effects on the neutral component resulting from the presence of magnetic field, Darcy's effect and the fields of gravity and pressure are neglected.

Let  $p, \rho, T, \alpha, \kappa, \kappa' \left( = \frac{\kappa'}{\rho c_v} \right), g, \mu, \nu \left( = \frac{\mu}{\rho} \right), v_c, \eta, k_1, q(u, v, \omega)$  and  $q_d(l, r, s)$

denote respectively, the pressure, density, temperature, thermal coefficient of expansion, thermal conductivity, thermal diffusivity, gravitational acceleration, viscosity, kinematic viscosity, collisional frequency between two components of composite medium, resistivity, medium permeability, velocity of ionized gas and velocity of neutral gas. Then the equations expressing the conservation of momentum, mass, heat and equation of state for the compressible, composite plasma layer are

$$\begin{aligned} \frac{\rho}{\varepsilon} \left[ \frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon} (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = - \nabla p + g\rho + \frac{1}{4\pi} (\nabla \times \mathbf{H}) \times \mathbf{H} \\ + \frac{\rho_d v_c}{\varepsilon} (\mathbf{q}_d - \mathbf{q}) - \frac{\rho \nu}{k_1} \mathbf{q}, \end{aligned} \quad (1)$$

$$\varepsilon \frac{\partial \rho}{\partial t} + (\mathbf{q} \cdot \nabla) \rho + \rho \nabla \cdot \mathbf{q} = 0, \quad (2)$$

$$E \frac{\partial T}{\partial t} = \beta \omega + \kappa \nabla^2 T, \quad (3)$$

$$\rho = \rho_0 [1 - \alpha(T - T_0)], \quad (4)$$

where  $\rho_0, T_0$  are density and temperature at the lower boundary  $z = 0$ . The kinematic viscosity  $\nu$  and the thermal diffusivity  $\kappa$  are assumed to be constants. We have taken the cartesian coordinates  $(x, y, z)$  with the origin on the lower boundary  $z = 0$  and  $z$ -axis perpendicular to it along the vertical. In the equation of motion for the neutral component, there will be an equal and opposite term to that in the equation of motion (1) for ionized component and so

$$\frac{\partial \mathbf{q}_d}{\partial t} + \frac{1}{\varepsilon} (\mathbf{q}_d \cdot \nabla) \mathbf{q}_d = - v_c (\mathbf{q}_d - \mathbf{q}). \quad (5)$$

The Maxwell's equations yield

$$\varepsilon \frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{q} \times \mathbf{H}) + \varepsilon \eta \nabla^2 \mathbf{H}, \quad (6)$$

$$\nabla \cdot \mathbf{H} = 0, \quad (7)$$

where  $E = \varepsilon + (1 - \varepsilon)(\rho_s c_s / \rho c)$  is a constant.  $\rho_s, c_s$  and  $\rho, c$  stand for density and specific heat of solid (porous matrix) and ionized component, respectively.

The initial state is

$\mathbf{q} = (0, 0, 0), T = T(z), p = p(z), \rho = \rho(z), \mathbf{q}_d = (0, 0, 0), \mathbf{H} = (0, 0, H)$  where, following Spiegel and Veronis [2], we have

$$\left. \begin{aligned}
 T(z) &= -\beta z + T_0, \\
 p(z) &= p_m - g \int_0^z (\rho_m + \rho_0) dz, \\
 \rho(z) &= \rho_m \left[ 1 - \alpha_m (T - T_m) + K_m (p - p_m) \right], \\
 \alpha_m &= - \left[ \frac{1}{\rho} \frac{\partial \rho}{\partial T} \right]_m \quad (= \alpha, \text{ say}) \\
 K_m &= \left[ \frac{1}{\rho} \frac{\partial \rho}{\partial p} \right]_m.
 \end{aligned} \right\} \quad (8)$$

Spiegel and Veronis [2] expressed any state variable, say  $X$ , in the form

$$X = X_m + X_0(z) + X'(x, y, z, t),$$

where  $X_m$  stands for the constant space distribution of  $X$ ,  $X_0$  is the variation in  $X$  in the absence of motion and  $X'(x, y, z, t)$  stands for the fluctuations in  $X$  due to the motion of the fluid.  $\rho_m$  and  $p_m$  thus stand for the constant space distribution of  $\rho$  and  $p$  and  $T_0$ ,  $\rho_0$  stand for the temperature and the density of the fluid at the lower boundary  $z = 0$ .

Consider a small perturbation on the initial state and let  $\delta p$ ,  $\delta \rho$ ,  $\theta$ ,  $h(h_x, h_y, h_z)$ ,  $q(u, v, \omega)$  and  $q_d(l, r, s)$  denote respectively, the perturbations in pressure, density, temperature, magnetic field, ionized component velocity, and neutral component velocity. The change in density  $\delta \rho$ , caused mainly by the perturbation  $\theta$  in temperature, is given by

$$\delta \rho = -\alpha \rho_m \theta. \quad (9)$$

Then the linearized hydromagnetic perturbation equations, under Spiegel and Veronis [2] approximations and findings, are

$$\begin{aligned}
 \frac{\rho_m}{\epsilon} \frac{\partial q}{\partial t} &= -\nabla \delta p + g \delta \rho + \frac{1}{4\pi} (\nabla \times \mathbf{h}) \times \mathbf{H} + \frac{\rho_d v_c}{\epsilon} (q_d - q) \\
 &\quad \rho_m \mathbf{v} \cdot \mathbf{q},
 \end{aligned} \quad (10)$$

$$\nabla \cdot \mathbf{q} = 0, \quad (11)$$

$$\frac{\partial q_d}{\partial t} = -v_c (q_d - q), \quad (12)$$

$$E \frac{\partial \theta}{\partial t} = \left( \beta - \frac{g}{c_p} \right) \omega + \kappa \nabla^2 \theta, \quad (13)$$

$$\nabla \cdot \mathbf{h} = 0, \quad (14)$$

$$\varepsilon \frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{q} \times \mathbf{H}) + \varepsilon \eta \nabla^2 \mathbf{h}, \quad (15)$$

where  $g/c_p$  stands for the adiabatic gradient.

If we analyze the disturbances in terms of normal modes, we assume that the perturbation quantities are of the form

$$[\omega, \theta, h_z] = [W(z), \Theta(z), K(z)] \exp(ik_x x + ik_y y + nt), \quad (16)$$

where  $k_x$  and  $k_y$  are the wave numbers in the  $x$ - and  $y$ -directions,  $k = (k_x^2 + k_y^2)^{1/2}$  is the resultant wave number, and  $n$  is the growth rate.

Expressing the coordinates  $x, y, z$  in the new unit of length  $d$  and putting  $a = kd$ ,  $\sigma = nd^2/\nu$ ,  $p_1 = \nu/\kappa$ ,  $p_2 = \nu/\eta$ ,  $P_l = k_l/\varepsilon d^2$ ,  $\alpha_0 = \rho_d/\rho_m$  and  $D = \frac{d}{dz}$ . Eqs. (10)-(15), with the help of (9) and expression (16) in nondimensional form become

$$\left[ \sigma \left( 1 + \frac{\alpha_0 \nu_c d^2/\nu}{\sigma + \nu_c d^2/\nu} \right) + \frac{1}{P_l} \right] (D^2 - a^2)W + \frac{g d^2 a^2 \varepsilon}{\nu} \alpha \Theta - \frac{H d \varepsilon}{4 \pi \rho_m \nu} (D^2 - a^2)DK = 0, \quad (17)$$

$$(D^2 - a^2 - E p_1 \sigma) \Theta = - \frac{d^2}{\kappa} \left( \beta - \frac{g}{c_p} \right) W, \quad (18)$$

$$(D^2 - a^2 - p_2 \sigma) K = - \left( \frac{H d}{\eta \varepsilon} \right) D W. \quad (19)$$

If we eliminate  $\Theta$  and  $K$  between eqs. (17)-(19), we get

$$\begin{aligned} (D^2 - a^2) (D^2 - a^2 - E p_1 \sigma) \left[ (D^2 - a^2 - p_2 \sigma) \left\{ \sigma \left( 1 + \frac{\alpha_0 \nu_c d^2/\nu}{\sigma + \nu_c d^2/\nu} \right) + \frac{1}{P_l} \right\} + Q D^2 W \right] \\ = \varepsilon R \left( \frac{G-1}{G} \right) a^2 (D^2 - a^2 - p_2 \sigma) W, \end{aligned} \quad (20)$$

where  $G = \frac{c_p \beta}{g}$ ,  $R = g \alpha \beta d^4 / \nu \kappa$  is the Rayleigh number and  $Q = H^2 d^2 / 4 \pi \rho_m \nu \eta$  is the Chandrasekhar number.

Consider the case in which both the boundaries are free and the medium adjoining the fluid is non-conducting. The boundary conditions appropriate for the problem are

$$\left. \begin{array}{l} W = D^2 W = 0, \Theta = 0, X = 0 \\ \text{and } h \text{ are continuous} \end{array} \right\} \text{ at } z = 0 \text{ and } 1. \quad (21)$$

In the absence of any surface current, the tangential components of the magnetic field are continuous. Hence the boundary conditions, in addition to (21) are

$$DK = 0 \text{ on the boundaries.} \quad (22)$$

Using the above boundary conditions, it can be shown that all the even order derivatives of  $W$  must vanish for  $z = 0, 1$  and hence the proper solution of  $W$  characterizing the lowest mode is

$$W = W_0 \sin \pi z, \quad (23)$$

where  $W_0$  is a constant. Substituting (23) in (20), we obtain the dispersion relation

$$R_1 = \varepsilon^{-1} \left( \frac{G}{G-1} \right) \left[ \frac{(1+x) \left( 1+x+E p_1 \frac{\sigma}{\pi^2} \right) \left[ \left( 1+x+p_2 \frac{\sigma}{\pi^2} \right) \left\{ \frac{\sigma}{\pi^2} \left( 1 + \frac{\alpha_0 v_c d^2/v}{\sigma + v_c d^2/v} \right) + \frac{1}{P_l} \right\} + Q_1 \right]}{x \left( 1+x+p_2 \frac{\sigma}{\pi^2} \right)} \right], \quad (24)$$

where  $x = a^2/\pi^2$ ,  $R_1 = R/\pi^4$ ,  $Q_1 = Q/\pi^2$  and  $P = \pi^2 P_l$ .

### 3. The case of stationary convection

When the instability sets in as stationary convection, the marginal state will be characterized by  $\sigma = 0$ , and eq. (24) reduces to

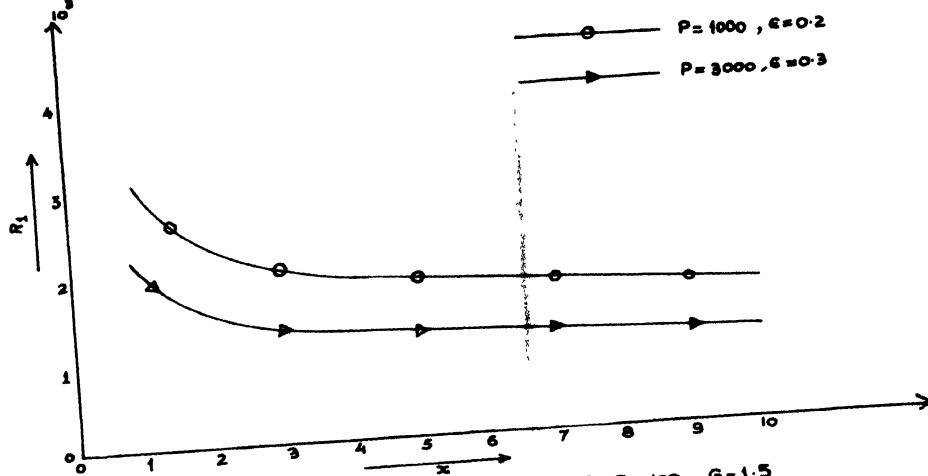
$$R_1 = \varepsilon^{-1} \left( \frac{G}{G-1} \right) \left[ \left( \frac{1+x}{x} \right) \left\{ \frac{1+x}{P} + Q_1 \right\} \right]. \quad (25)$$

This expresses the modified Rayleigh number  $R_1$  as a function of the dimensionless wave number  $x$  and the parameter  $P$  and  $Q_1$ . For fixed values of  $P$  and  $Q_1$ , let the nondimensional number  $G$  accounting for the compressibility effects be also kept as fixed. Then we find that

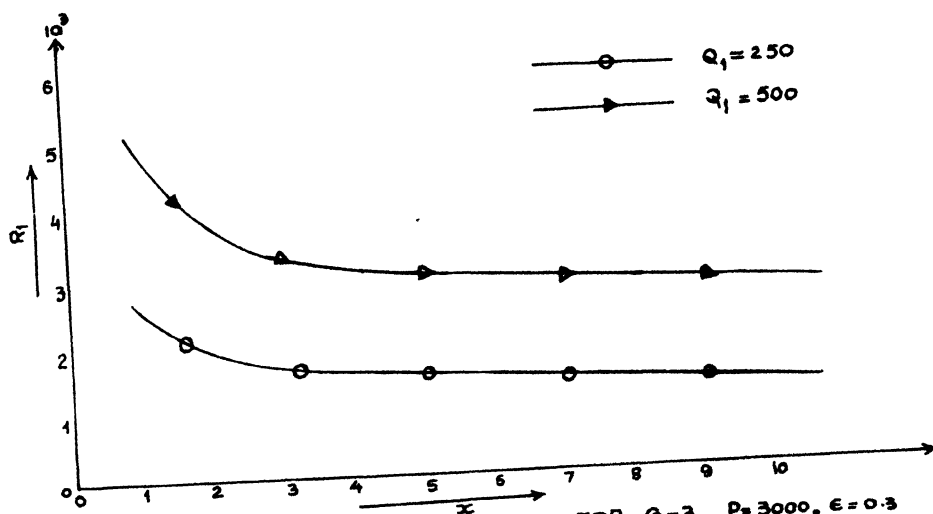
$$\bar{R}_c = \varepsilon^{-1} \left( \frac{G}{G-1} \right) R_c, \quad (26)$$

where  $\bar{R}_c$  and  $R_c$  denote respectively, the critical Rayleigh numbers in the presence and absence of compressibility. The effect of compressibility is, thus, to postpone the onset of thermal instability. The cases  $G < 1$  and  $G = 1$  correspond to negative and infinite values of critical Rayleigh numbers in the presence of compressibility, which are not relevant for the

present problem. Hence, we obtain a stabilizing effect of compressibility. It is evident from (25) that



THE VARIATION OF  $R_1$  WITH  $x$  FOR  $Q_1 = 100$ ,  $G = 1.5$   
Figure 1. The variation of  $R_1$  with  $x$  for  $Q_1 = 100$ ,  $G = 1.5$ .



THE VARIATION OF  $R_1$  WITH  $x$  FOR  $G = 3$ ,  $P = 3000$ ,  $\epsilon = 0.3$   
Figure 2. The variation of  $R_1$  with  $x$  for  $G = 3$ ,  $P = 3000$ ,  $\epsilon = 0.3$ .

$$\frac{dR_1}{dQ_1} = \left( \frac{G}{G-1} \right) \left( \frac{1+x}{\epsilon x} \right), \quad (27)$$

and

$$\frac{dR_1}{dP} = - \left( \frac{G}{G-1} \right) \left( \frac{1+x}{\epsilon x P^2} \right), \quad (28)$$

which imply that the magnetic field has a stabilizing effect whereas the medium permeability has destabilizing effect on the system.

We now give some realistic values to various parameters in eq. (25) to demonstrate the results through graphs. Figure 1 plots Rayleigh number  $R_1$  against wave numbers  $x$  for fixed values of  $Q_1 = 100$ ,  $G = 1.5$  and variable values of  $\varepsilon = .2$ ,  $P = 1000$  and  $\varepsilon = .3$ ,  $P = 3000$  respectively. It is clear from Figure 1 that medium permeability has destabilizing effect on the system. Figure 2 depicts  $R_1$  against  $x$  for fixed values of  $\varepsilon = .3$ ,  $P = 3000$ ,  $G = 3$  and variable values of  $Q_1 = 250$  and  $500$  respectively. Figures 1 and 2 demonstrate the stabilizing effects of compressibility and uniform magnetic field.

#### 4. Stability of the system and non-oscillatory modes

Multiplying (17) by  $W^*$ , the complex conjugate of  $W$ , integrating over the range of  $z$  and using (18) and (19) together with the boundary conditions (21) and (22), we obtain

$$\left[ \sigma \left( 1 + \frac{\alpha_0 v_c d^2/v}{\sigma + v_c d^2/v} \right) + \frac{1}{P_l} \right] I_1 + \frac{c_p \alpha \kappa a^2 \varepsilon}{v(1-G)} (I_2 + E p_1 \sigma^* I_3) + \frac{\eta \varepsilon^2}{4\pi \rho_m v} (I_4 + p_2 \sigma^* I_5) = 0, \quad (29)$$

where

$$\begin{aligned} I_1 &= \int_0^1 (|DW|^2 + a^2|W|^2) dz, \\ I_2 &= \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2) dz, \\ I_3 &= \int_0^1 (|\Theta|^2) dz, \\ I_4 &= \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2) dz, \\ I_5 &= \int_0^1 (|DK|^2 + a^2|K|^2) dz, \end{aligned} \quad (30)$$

where  $\sigma^*$  is the complex conjugate of  $\sigma$  and the integrals  $I_1 - I_5$  are all positive definite. Putting  $\sigma = \sigma_r + i\sigma_i$  and then equating the real and imaginary parts of (29), we obtain

$$\left[ \frac{(\sigma_r + v_c d^2/v)(\sigma_r^2 + \sigma_r(1 + \alpha_0)v_c d^2/v) + \sigma_i^2(\sigma_r + \alpha_0 v_c d^2/v)}{[(\sigma_r + v_c d^2/v)^2 + \sigma_i^2]} + \frac{1}{P_l} \right] I_1$$



$$+ \frac{c_p \alpha \kappa a^2 \epsilon}{v(1-G)} (I_2 + Ep_1 \sigma_r I_3) + \frac{\eta \epsilon^2}{4\pi \rho_m v} (I_4 + p_2 \sigma_r I_5) = 0 \quad (31)$$

and

$$i\sigma_i \left[ \frac{\sigma_r^2 + \sigma_i^2 + 2\sigma_r v_c d^2/v + (1+\alpha_0) (v_c d^2/v)^2}{\left[ (\sigma_r + v_c d^2/v)^2 + \sigma_i^2 \right]} I_1 + \frac{c_p \alpha \kappa a^2 \epsilon}{v(G-1)} Ep_1 I_3 - \frac{\eta \epsilon^2}{4\pi \rho_m v} p_2 I_5 \right] = 0. \quad (32)$$

It follows from (32) that if  $G > 1$  and if the magnetic field is absent,  $\sigma_i = 0$ . This means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for a porous medium in the absence of magnetic field. This result is true for the porous as well as non-porous medium. The presence of magnetic field brings oscillatory modes (as  $\sigma_i$  may not be zero) which were non-existent in its absence.

### 5. The case of overstability

Here we consider the possibility of whether instability may occur as overstability. Put  $\sigma/\pi^2 = i\sigma_i$ ,  $\sigma$  may be complex. Since for overstability, we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (24) will admit of solutions with  $\sigma_i$  real. Eq. (24) becomes

$$R_1 = \epsilon^{-1} \left( \frac{G}{G-1} \right) \times \left[ \frac{(1+x)(1+x+iEp_1\sigma_i) \left[ (1+x+ip_2\sigma_i) \left\{ i\sigma_i \left( 1 + \frac{\alpha_0 v_c d^2/v}{i\sigma_i \pi^2 + v_c d^2/v} \right) + \frac{1}{P} \right\} + Q_1 \right]}{x(1+x+ip_2\sigma_i)} \right]. \quad (33)$$

If we equate real and imaginary parts of (33) and eliminate  $R_1$  between them, we obtain

$$A_2 C_1^2 + A_1 C_1 + A_0 = 0, \quad (34)$$

where we have put  $C_1 = \sigma_i^2$ ,  $b = 1+x$  and

$$A_2 = bp_2^2 P^2 \pi^4 \left[ b + Ep_1 \left( \frac{\alpha_0 v_c d^2}{\pi^2 v} + \frac{1}{P} \right) \right],$$

$$A_1 = P^2 \pi^4 \left[ b^4 + \left\{ Ep_1 \left( \frac{\alpha_0 v_c d^2}{\pi^2 v} + \frac{1}{P} \right) \right\} b^3 + \left\{ p_2^2 (1 + \alpha_0) \left( \frac{v_c d^2}{\pi^2 v} \right)^2 \right. \right. \\ \left. \left. + Q_1 (Ep_1 - p_2) \right\} b^2 + \left\{ \frac{Ep_1 p_2^2}{P} \left( \frac{v_c d^2}{\pi^2 v} \right)^2 \right\} b \right],$$

and

$$A_0 = P^2 \pi^4 \left[ \left\{ (1 + \alpha_0) \left( \frac{v_c d^2}{\pi^2 v} \right)^2 \right\} b^4 + \left\{ \frac{Ep_1}{P} \left( \frac{v_c d^2}{\pi^2 v} \right)^2 \right\} b^3 \right. \\ \left. + \left\{ Q_1 (Ep_1 - p_2) \left( \frac{v_c d^2}{\pi^2 v} \right)^2 \right\} b^2 \right]. \quad (35)$$

Since  $\sigma_1$  is real for overstability, both the values of  $C_1 (= \sigma_1^2)$  are positive. Eq. (34) shows that this is impossible if  $A_2 > 0$ ,  $A_1 > 0$  and  $A_0 > 0$ . So  $A_2 > 0$ ,  $A_1 > 0$  and  $A_0 > 0$  yield the sufficient conditions for the non-existence of overstability, which give

$$Ep_1 \geq p_2, \quad (36)$$

i.e.  $\kappa \leq \eta E$ . (37)

Hence  $\kappa \leq \eta E$  is the sufficient condition for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

We examine the natures of  $dR_1/dQ_1$  and  $dR_1/dP$  to study the effect of a magnetic field and medium permeability, respectively, on the thermal instability of a compressible and composite plasma in porous medium for the overstable case. It follows from eq. (33) that

$$\frac{dR_1}{dQ_1} = \varepsilon^{-1} \left( \frac{G}{G-1} \right) \left( \frac{1+x}{x} \right) \left[ \frac{(1+x)^2 + Ep_1 p_2 \sigma_1^2 + i \sigma_1 (1+x) (Ep_1 - p_2)}{[(1+x)^2 + p_2^2 \sigma_1^2]} \right]. \quad (38)$$

The imaginary part of (38) equated to zero gives

$$Ep_1 = p_2. \quad (39)$$

Equating the real parts of (38) and substituting (39) in it, we get

$$\frac{dR_1}{dQ_1} = \left( \frac{G}{G-1} \right) \left( \frac{1+x}{\varepsilon x} \right). \quad (40)$$

Eq. (40) implies that  $dR_1/dQ_1$  is positive if  $G > 1$ . Hence for  $G > 1$ , the Rayleigh number increases as the magnetic field increases, showing the stabilizing effect of magnetic field. It is evident from eq. (33) that

$$\frac{dR_1}{dP} = -\left(\frac{G}{G-1}\right)\left(\frac{1+x}{\epsilon x P^2}\right)(1+x+iE p_1 \sigma_1). \quad (41)$$

Using the imaginary part of (41) equated to zero in the real part of (41), we get

$$\frac{dR_1}{dP} = -\left(\frac{G}{G-1}\right)\frac{(1+x)^2}{\epsilon x P^2}, \quad (42)$$

which implies that medium permeability has a destabilizing effect on the system.

## 6. Conclusion

A compressible and partially ionized plasma layer heated uniformly from below in porous medium, is of quite frequent occurrence and importance in cosmic physics *e.g.* in the solar photosphere and chromosphere and in cool interstellar clouds and in geophysics. The magnetic field postpones whereas the medium permeability hastens the onset of thermal convection for both stationary and overstable cases. Magnetic field introduces oscillatory modes in the system which were non-existent in the absence of magnetic field. A sufficient condition for the non-existence of overstability has been obtained.

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